Sets

A **set** is an unordered collection of distinct objects. (members or **elements** of the set)

Exs: $\{1, 5, 13\}$

$\{a, 2, \Delta, ☺\}$

$\{\triangle, \triangle, \triangle, \triangle, \triangle, \triangle\}$

$\mathbb{N} = \mathbb{Z}^{+} = \{1, 2, 3, \ldots\}$ (**natural numbers**)

$\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$ (**integers**)

$\phi = \{\ \}$ (**empty set**)

Notation: If A is a set then

- $a \in A$ means "a is an element of A"
- $a \notin A$ means "a is not an element of A"

The **cardinality** of a finite set A is the number of elements in A.

Notation: $|A|$, $\# A$, $\operatorname{card}(A)$

Exs: $|\{1, 5, 13\}| = 3$

$|\{\triangle, \triangle, \triangle, \triangle, \triangle, \triangle\}| = 6$

$|\phi| = 0$

If A is not finite then we write $|A| = \infty$.

Two ways of describing sets:

① Set roster notation:

List out the elements of a set between curly braces.

Exs: (all previous examples)

A couple of trickier examples:

- $A = \{1, \{1\}\}$

  elements of A: 1 and $\{1\}$ $(|A| = 2)$

- $A = \{\phi, 2, \{a, b\}, \{1, \{1\}\}\}$

  elements of A: ϕ, 2, $\{a, b\}$, $\{1, \{1\}\}$ $(|A| = 4)$

② Set builder notation:

Specify a collection of elements which satisfy certain conditions.

Notation: $\{\text{elements} : \text{conditions}\}$

-or-

$\{\text{elements} \mid \text{conditions}\}$

Exs:

$\mathbb{Z}_{\geq 0} = \{n \in \mathbb{Z} : n \geq 0\}$ (non-negative integers)

(real numbers)

$\mathbb{Q} = \{x \in \mathbb{R} : x = \frac{p}{q} \text{ for some } p \in \mathbb{Z},\ q \in \mathbb{N}\}$

(rational numbers)

$\{x \in \mathbb{R} : x \notin \mathbb{Q}\}$ (irrational numbers)

Subsets

If A and B are sets then we say that A is a <u>subset</u> of B if every element of A is also an element of B.

Notation:

- $A \subseteq B$ means "A is a subset of B"
- $A \nsubseteq B$ means "A is not a subset of B"

If $A \subseteq B$ and $A \neq B$ then we say that A is a <u>proper subset</u> of B.

Note: Two sets are equal if and only if they have exactly the same elements. Equivalently, $A = B$ if and only if $A \subseteq B$ <u>and</u> $B \subseteq A$.

ex: $\frac{1}{2} \in \mathbb{Q}$ but $\frac{1}{2} \notin \mathbb{Z}$

Exs: • $\mathbb{Z} \subseteq \mathbb{Q}$, but $\mathbb{Q} \not\subseteq \mathbb{Z}$

ex: $\sqrt{2} \in \mathbb{R}$ but $\sqrt{2} \notin \mathbb{Q}$

$\mathbb{Q} \subseteq \mathbb{R}$, but $\mathbb{R} \not\subseteq \mathbb{Q}$

• For any set A, we have that

$$\emptyset \subseteq A \quad \text{and} \quad A \subseteq A$$

• Let $A = \{1, \{2\}\}$. Then

$\{1\} \subseteq A$, but $1 \not\subseteq A$,

and $\{\{2\}\} \subseteq A$, but $\{2\} \not\subseteq A$.

List of all subsets of A:

$$\emptyset, \ \{1\}, \ \{\{2\}\}, \ \underbrace{\{1, \{2\}\}}_{= A}.$$